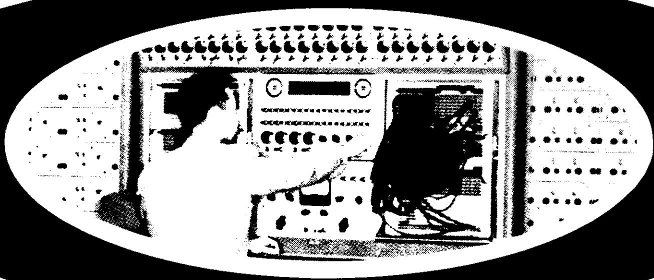
APPLICATION BULLETIN No. 3

Analogue Computers In Process Design



Analog Computer

APPLICATION BULLETIN

The **Key** *to Progressive Engineering*

Electronic Associates, Inc.

Manufacturers of PACE *Precision Analog Computing Equipment*

THE AUTHOR

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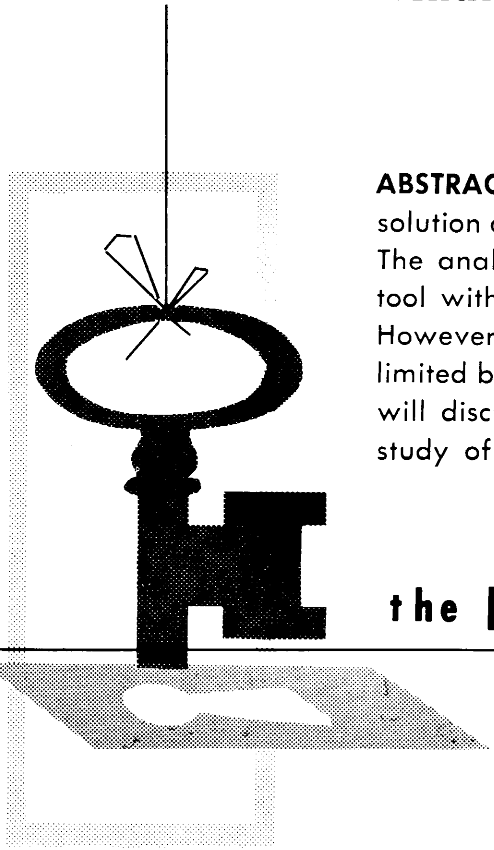
Originally employed as an Applications Engineer at Electronic Associates' Princeton Computation Center, Mr. Worley's work covered the development of industrial applications for the analog computer. He is a licensed professional engineer and has authored several technical papers on the subject of process control.

In his present capacity as Manager of Market Development, he is concerned with the use of the Analog Computer in solving engineering and economic problems associated with the automization of industrial processes.

Analog Computers in Process Design

ABSTRACT — The analysis of industrial processes requires the numerical solution of a large number of linear and non-linear different equations. The analog computer provides the process engineer with a practical tool with which these solutions can be quickly and easily obtained. However, use of analog computers for process analysis is presently limited by lack of familiarity with them by many engineers. This paper will discuss the use of the general purpose analog computer in the study of process control systems and design of industrial processes.



the key to progressive engineering

Introduction

The utility of the analog computer in the aircraft and missile industries is an accepted fact. Almost all major aircraft companies have large analog computer installations which have proven to be extremely valuable for the solution of problems associated with the design of modern aircraft and guided missiles.

The use of the general purpose analog computer in the processing industries has increased tremendously during the past two years and indications are that they can become an even more valuable tool for the solution of problems in the processing industries. This seems obvious to the thinking engineer since problems associated with the various branches of engineering are basically the same. The problems associated with the guidance of a ballistic missile are essentially the same as that of the control of an industrial process.

Theory of Operation

It is of interest to note that the basic theory of operation and application of the modern analog computer is not new. The similarity of the mathematical laws which govern mechanical motion, heat transfer, fluid flow, chemical kinetics, and the flow of electrical current serves as a basis upon which analogies can be made between different physical systems. Early papers by Baker and Pachkis[1] have done much to develop the direct analogy method of obtaining analytical and experimental solutions to enginering problems. Although the direct analog has been and will continue to be of use to the system engineer in visualizing complete physical systems, its use as a general purpose equation solver suffers due to some rather serious problems. Passive element analogs of complete physical systems require rather complex electrical circuits and consequently it becomes difficult to eliminate purely electrical circuit problems. There is also the problem of sizing and selection of circuit parameters to fit the physical system being investigated. For this reason active circuit elements, i.e. operational amplifiers, are used in the analog computer of today.

The modern analog computer depends upon only one analogous relationship and that is the integrating characteristic of the charge on an electrical capacitor. Thus, the analog computer performs basically as a mathematical equation solver. Yet it retains the capabilities of direct simulation which is its biggest attraction to the engineer.

"This paper presented at ISA 1958 National Symposium for Chemical and Petroleum Instrumentation, February 3 and 4, 1958, Wilmington, Delaware."

The general purpose analog computer has the ability to handle a wide variety of engineering problems, all expressible in terms of differential or algebraic equations. Unlike digital computers, analog computers employ distinct computing elements for each mathematical operation required to solve a given problem. This type of operation, known as parallel operation, is an essential reason for the very high computing speeds possible with the d-c analog computer; most problems, regardless of complication, are solved within seconds by general purpose analog computers and within fractions of a second by repetitive machines. Parallel operation, however, imposes some practical limits on the complexity of problems which can be solved on analog computers. The average problem solved today employs from 15 to 40 integrations, 20 to 60 summations, 10 to 25 multiplications and divisions of two variables, and from 2 to 10 function generations. Problems involving from 40 to 150 integrations, 60 to 400 summations, 25 to 125 multiplications and divisions of two variables, and from 10 to 50 function generations are not uncommon. Accuracies obtained vary from 0.05% to 0.50% depending upon the components used and the characteristics and complexity of the problem. Basic component accuracies of the modern analog computer are 0.01%.

Accordingly, it is customary to obtain more complicated mathematical operations through combinations of a limited number of simple operations, performed by basic computing elements. It has been proved explicitly[2] that a wide range of problems can be solved conveniently by the application of only the following computing elements:

1. Devices which multiply by positive or negative constant coefficients.
2. Devices which sum two (or more) variables.
3. Devices which produce the product of two variables.
4. Devices to generate functions of variables.
5. Devices that generate the time integral of a variable.

The machine variables which these components operate on are d-c voltages made proportional to the variables of given problems. The inputs and outputs of each of these components are collected at a central "patch board" on the computer console. Plug-in "patch cords" are used to connect the components as specified by the equations to be solved. The conveniently measurable computer voltages will then vary so that the records of their values or behavior constitute solutions of the given problem.

Although the analog computer utilizes electronic components and electrical circuit characteristics in its operation, it is not essential that the analog computer user have an extensive knowledge of electrical circuits. In fact, experience has shown that any good engineer can become proficient in the use of the analog computer in a relatively short time; often in a matter of a few weeks. With the analog technique it is only necessary to translate the problem equations into a series of connections between the standard computer components. These connections are shown on a computer diagram which results from programming the problem. For simplification in programming, the computer components are represented by symbols, characteristic of the mathematical functions performed by the component, which can be thought of as mathematical building blocks. The resulting computer diagram becomes in effect a detailed signal flow diagram of the problem being solved. The analog computer symbols in most common use are shown in Table I.[3]

Analog Solution of Differential Equations

The type of problems best adapted to solution on an analog computer are those involving systems of simultaneous differential equations, linear or non-linear, with constant or varying coefficients. Fortunately, the complexity of problem set-up is increased only slightly for non-linear problems and problems involving non-constant coefficients. Problems other than those which belong in the category of ordinary differential equations can also be satisfactorily solved with an analog computer.

The solution of differential equations with the analog computer is based upon the mathematical technique of repeated integration. The use of the method of successive integration is illustrated by the following sample problem. Consider a spring-loaded pneumatically operated diaphragm motor used for operating control valves. Oldenbourg and Sartarius[4] have shown that the equation of this arrangement is

$$M \frac{d^2x}{dt^2} + D \frac{dx}{dt} + Kx = F(t) \qquad (1)$$

Solving the equation for the highest derivative

$$\frac{d^2x}{d\,t^2} = \frac{F(t)}{M} - \frac{D}{M}\frac{dx}{dt} - \frac{Kx}{M} \qquad (2)$$

Equation (2) states that if voltages proportional to F(t)M, (D/M)/(dx/dt), and Kx/M, are summed the resulting voltage will be proportional to the second derivative or valve travel. Integrating this voltage gives the first derivative of valve travel. This second voltage when integrated yields the valve travel x. Thus the problem variables represented by computer voltages required for forming the second derivative of valve travel have now been developed. These voltages are multiplied by the system parameters and scaling constants and then summed and integrated in the first integrator. The complete computer diagram for solving this equation is shown in Figure 1.

These simple programming techniques, although here applied to a relatively simple system, are the same as those used in programming more complex systems. The ease in programming is evident. Each equation or physical component may be treated separately, hence, more complicated mathematical operations can be performed through combinations of a limited number of simple operations. Thus the analog computer can, in effect, simulate the operation of large physical systems since it combines and simultaneously solves the many equations representing its behavior.

Applications To Industrial Processes

Because of the nature of their operation d-c analog computers are particularly well suited for engineering

EQUATION:

$$M \frac{d^2x}{dt^2} + D \frac{dx}{dt} + KX = F(t)$$

COMPUTER CIRCUIT:

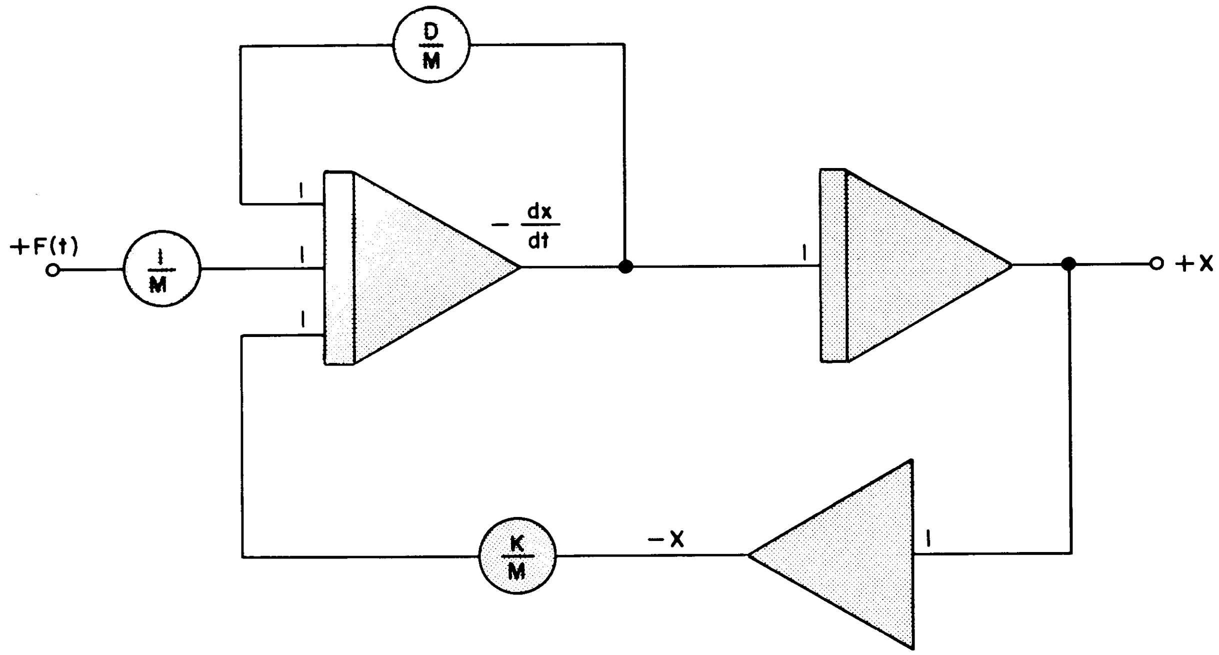


FIGURE I. ANALOG SIMULATION OF A PNEUMATIC CONTROL VALVE MOTOR

analysis since they lend themselves readily to the solution of the mathematical expressions describing the performance of physical systems. A realistic appraisal of analog computation techniques should point out, however, that the analog computer must be regarded only as a tool for the solution of engineering problems. Since the mechanics[5] of their use are relatively straight forward and easily visualized any discussion of computer applications must deal primarily with derivation of equations used to describe physical systems. This is a difficult subject to accommodate since the techniques of analysis of physical systems encompass the entire field of engineering practices and fundamentals. Consequently, this discussion will primarily attempt to point out areas where analog computers have been used with success in the design of industrial processes and control systems.

Their use in solving engineering problems in the processing industries will generally fall into two areas of application: (1) Control System Design, and (2) Applied Research and Processing Design.

Control System Design

Perhaps the most extensive and fruitful applications have been in the field of automatic control engineering. D-c analog computing elements lend themselves naturally to the representation of feedback loops analogous to those used in control systems, and the analog nature of the computer input and output data permits one to introduce components of actual systems into the feedback loops for system tests.

In the design of process control systems one of the questions to decide is which of the many types of control systems is likely to give satisfactory operation. Ideally each control system should be designed as a complete unit to produce the required quality of control at the least initial and running costs. In the recent past this has not been done because the potential economic advantages of such a design either could not or was not evaluated. It has been normal practice for the designer of the plant or process to call for certain standard types of control equipment, taking the advice of the instrument manufacturer on their suitability for a given application, and to depend on the flexibility of such equipment to permit its adjustment to meet the plant requirements.

It has become increasingly apparent to many industrial organizations[6] that this approach does not always result in satisfactory performance of the process. They

THEORETICAL EQUATION:

$$P_0(t) = K_c\varepsilon + \frac{K_c}{\tau_c}\int \varepsilon \cdot dt$$

COMPUTER CIRCUIT:

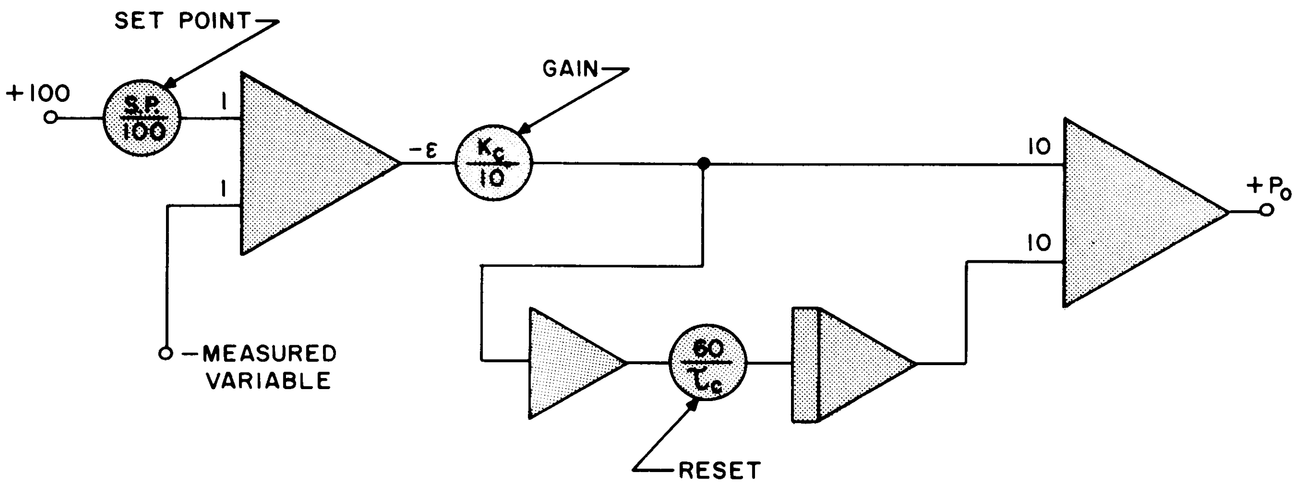
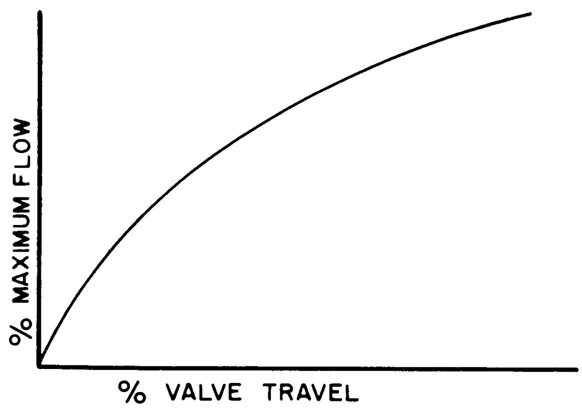


FIGURE 2. ANALOG SIMULATION OF A THEORETICAL PROPORTIONAL PLUS RESET CONTROLLER

VALVE CHARACTERISTICS:



COMPUTER CIRCUIT:

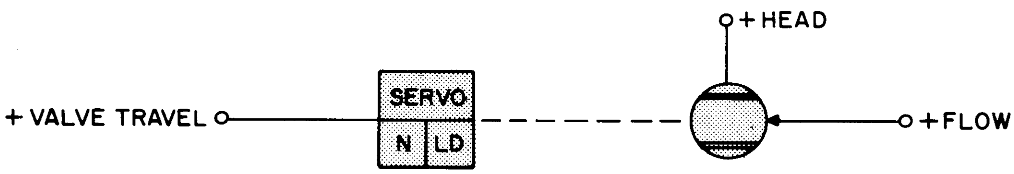


FIGURE 3: ANALOG SIMULATION OF CONTROL VALVE CHARACTERISTICS.

TRANSFER FUNCTION:

$$\frac{T_o}{T_i}(p) = \frac{K}{(\tau_1 p + 1)(\tau_2 p + 1)}$$

COMPUTER CIRCUIT:

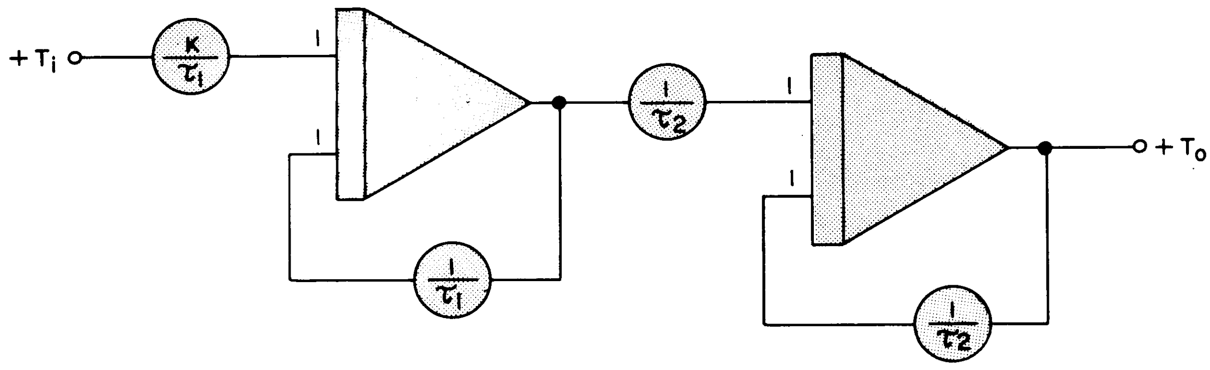


FIGURE 4. ANALOG SIMULATION OF JACKETED THERMOCOUPLE

are beginning to realize that the mere fact that a system is working is not sufficient criterion for qualification as a suitable control system. Consequently, control systems for process requiring speed and precision of control are presently being designed with the aid of the so-called system engineering techniques. These techniques, based on the theory of feedback control, offer a systematic procedure for a consideration of the dynamic behavior of all components in the system, including the process. The study of the dynamic behavior of a closed loop necessitates the proper defining of the dynamic characteristics of all the elements constituting the loop. Since the study of the performance of physical devices involves a consideration of their energy transfer characteristics, the definition of the operation of control system components normally results in some form of differential equation.

Broadly speaking, therefore, what is required is a mathematical formulation of the measuring system, the control valve, the controller, and the process. The first three are rather easily definable since their mechanism, although complex in many cases, can be usually represented by well defined physical fundamentals. Consequently, the mathematical descriptions of instrumentation devices have become almost standardized and the usual practice in the analog simulation of process control systems is to use standard analog computer diagrams with parameters or constants depending upon the particular manufacturer. In Figure 1 the analog circuit for a pneumatic control valve operator was presented. Figures 2, 3, and 4 show the computer simulation for a theoretical proportional plus reset controller, a valve body, and jacketed thermocouple.

The process because of its complexity and general lack of basic data regarding its energy transfer mechanisms is generally much more difficult to define mathematically. The quantitative treatment of chemical processes is complicated because heat, momentum, and mass transfer frequently occur simultaneously with chemical reactions. This coupled with the complexity of analysing and correlating rate data for chemical reactions has hindered the development of mathematical expressions for defining chemical operations. Recent progress in the science of chemical kinetics plus better experimental techniques have led to significant progress in interpreting processes where chemical reactions are accompanied by physical transfer operations. This increased ability to define chemical process behavior has encouraged the use of electronic computers which has in turn further increased the understanding of process operation.

Initial attempts to define the controlability of industrial processes were based on the time lag properties of physical systems. Perhaps the first published attempt at understanding the dynamics of processes was in a paper by Ivanoff[7] in 1934, followed in 1936, by a joint effort by Callendar,[8] Hartree, and Porter who investigated the effects of time lag in a control system. The next step was the application of the Servomechanism techniques to process control systems discussed in 1950 by Rutherford.[9] In 1952, McMahon[10] and Ackley pointed out those process characteristics which affect automatic control. Since that time much work has been done in the application of these techniques to process control, and gradually a better understanding of the dynamic behavior of a wide variety of plants is being obtained.

These early papers, although doing much to help in an appreciation of the basic energy transfer mechanisms of industrial processes, primarily dealt with simple processes on a linear basis. Despite the simplicity of this treatment of processes equations difficult to solve manually were obtained when they were combined with instrumentation devices into a closed loop system. Hence, the services of a computer were required. Medkeff and Matthews[11] discuss the use of analog computers in solving these simplified process control problems.

Such control investigations were based on the assumption that the process lags are caused by capacitance, resistance, and dead time effects. Although the visualization and the sizing of these effects were based on a consideration of the transfer of energy, there was a growing realization that the energy mechanisms of physical processes should and could be considered on a more fundamental basis. Such an approach utilizes the basic laws of physics, chemistry and mathematics. Mathematics serve as the tool by means of which knowledge of physical and chemical principles can be applied.

Batke, Franks and James[12] describe the results of a more fundamental approach to the control of a large chemical reactor. Their approach was to divide the reactor into a number of physical zones. Heat and material balances were then made for each zone resulting in a series of differential equations, which when solved simultaneously gave the performance of the reactor. Although this approach is largely intuitive, it does have a theoretical basis. The theoretical approach would be to write the equalibrium equations with respect to the proper space dimensions. The application of finite difference techniques to the resulting partial differential equation produces exactly the same equations as solved by the authors.

As an illustration of this approach consider the transfer of heat by conduction in a heat exchanger. Howe[13] discussed the solution of heat conduction on the analog computer by the application of finite difference techniques.

To be theoretically accurate, heat flow, like the flow of electric charge, must be represented by a system which has parameters regarded as distributed throughout the body. For this reason "lumped" parameters are usually assumed by dividing the heat transfer surface into sections of uniform temperature. The mass in each section is assumed to be confined to the boundries of the section while all increases or decreases in temperature are considered to take place between the sections of uniform temperature. The accuracy of the analysis naturally depends upon the number of sections considered.

The mathematical justification for this approach is based upon the theory of equations of finite differences. Equations obtained from the application of the theory of finite differences to the general equation for the heat flow by conduction are essentially those arrived at by making a heat balance for each of the geometric sections of the heat exchange equipment. A heat balance equation for each section is a simple statement of the fact that

RATE OF HEAT STORED IN SECTION = HEAT FLOW IN—HEAT FLOW OUT (3)

Since rates of heat flow are involved in the heat balance, this equation usually takes the form of a simple differential equation. The resulting equation and the computer circuit required for solving it are shown in Figure 5. The output of the computer circuit will be a d-c voltage proportional to the average temperature in the nth section providing the correct scale factors are applied.

EQUATION:

$$\frac{M_p C_p}{\ell} \frac{dT_p^{n'}}{dt} = \frac{UA}{\ell} (T_s^n - T_p^{n'}) + 2\omega_p C_p (T_p^n - T_p^{n'})$$

COMPUTER CIRCUIT:

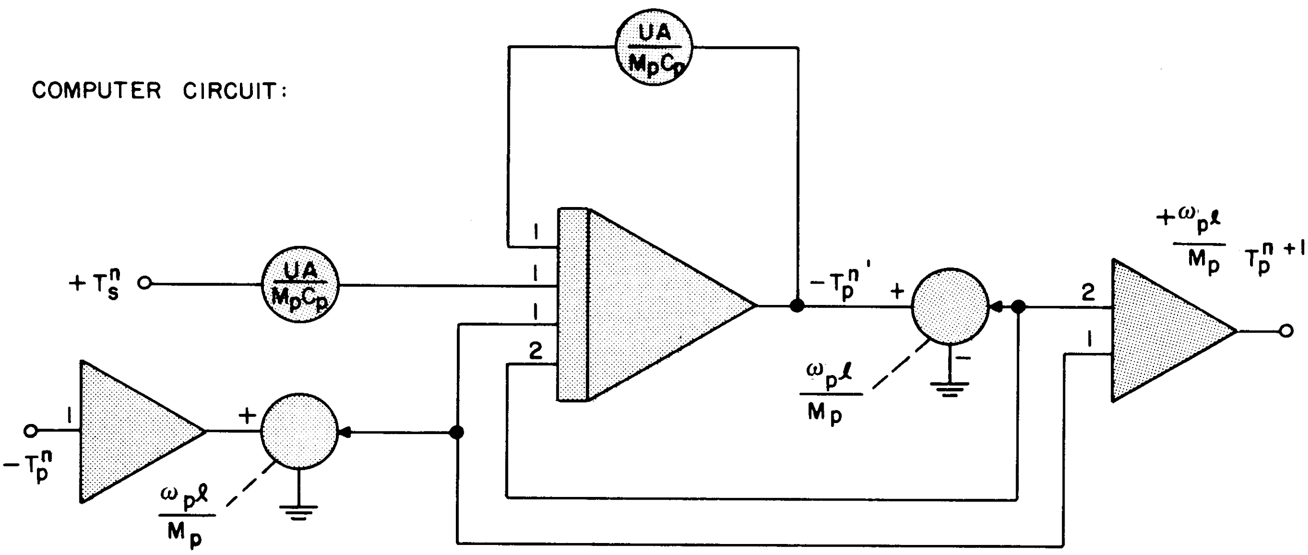


FIGURE 5. ANALOG SIMULATION OF THE n^{th} SECTION OF A HEAT EXCHANGER.

EQUATION:

$$L_{n+1} X_{n+1} = V_n Y_n + W X_W$$

COMPUTER CIRCUIT:

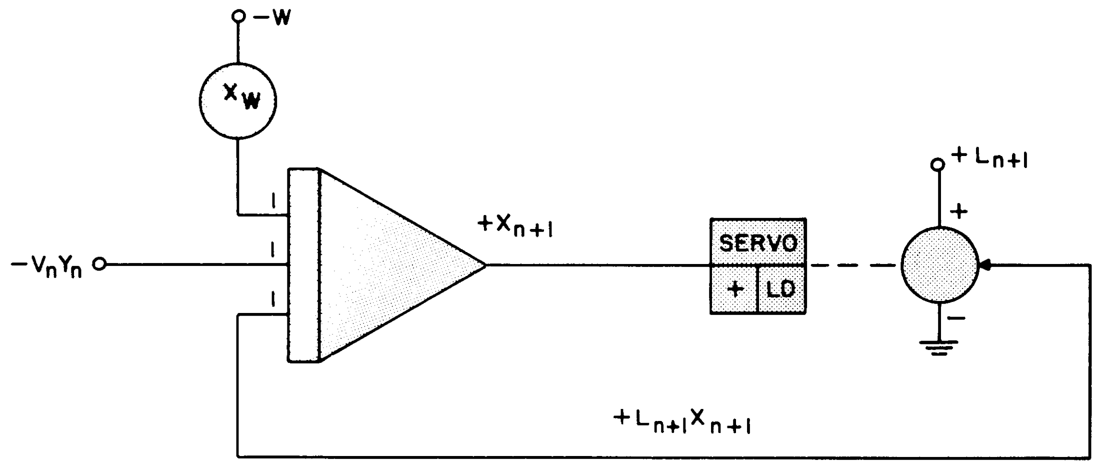


FIGURE 6. ANALOG SOLUTION OF MATERIAL BALANCE FOR BOTTOM PLATE OF A DISTILLATION COLUMN

Since the equation was developed from a basic energy balance it can, with some modifications, be applied to a variety of heat transfer problems. Should the process being analysed possess some form of chemical reaction, then additional mass balance equations for each section must be included. The conversion of mass by chemical reaction must of necessity be included. By application of these techniques exceedingly complex chemical processes can be analysed on a dynamical basis.

Unfortunately, for the engineers, the analysis of most physical systems, if carried out on a rigorius basis, results in some form of partial differential equation. The problem of solving such equations is not simple, no matter the method used. Solutions are usually restricted to simpler cases in two or possibly three dimensions. Fortunately, many engineering problems can be solved with these restrictions. The analog computer can be used to solve these simplified equations. For the two dimensional case, with linear or non-linear equations, the results are quite satisfactory with a reasonable amount of equipment. However, for three or more dimensions, the amount of equipment required becomes excessive and the problem is seldom attempted.

Again the accuracies obtained depend upon the number of sections used and hence the amount of equipment required. Fisher[14] shows that the accuracy of the finite difference method can be expressed as a function of the order of the difference and the number of sections.

For a second order difference
error $= (\Delta x)^2$

For a fourth order difference
error $= (\Delta x)^4$

Thus for a second order difference ten sections will result in a 1% error in the method. For a fourth order difference five sections will result in an error of 0.16%. One encouraging factor is that a higher order difference requires the same number of operational amplifiers as a second order difference. However, more coefficient potentiometers are required but they are relatively inexpensive computer components.

What results can be obtained from an investigation of process control systems with the analog computer?

Typical questions quickly answered are:

1. Will the proposed control system do a good job of control?
2. What is optimum control?
3. What are the controller settings which will give optimum control?
4. What is the best start-up procedure?
5. Are the pre-construction system specifications adequate?

The question naturally arises as to how this approach can improve control system design.

1. Different control schemes can be tried quickly and easily.
2. The simulated control system can be disturbed without upsetting production.
3. Safety limits can be evaluated without danger.
4. Efficient start-up procedure can be worked out.

5. Process and instrumentation parameters can be quickly changed.

Applied Research and Process Design

Because of the success of analog computers in solving control problems for the aircraft industry, it was generally thought that the use of the analog machine in the processing industries would be for control studies. Although the analog has indeed proven valuable for control studies, there is a definite trend toward more of a preponderence of applications in the design of processes.

The design of chemical processes requires a knowledge of those parameters affecting the process and the method of operation of the process. This information along with the required production of product constitutes the design conditions. They are the variables that must be chosen by the engineer before the design can be carried out. The optimum design is that which will be the most economical, i.e., that which will require the lowest total cost per unit of product. Included in the total cost will be initial design and construction expenditures and all operating costs. The pattern for the ideal approach to process design is apparent: first, the calculations should be carried out for a number of design conditions which are likely to result in low total costs and second, the optimum conditions are chosen from the result of these calculations. In the past this approach to design has not been possible because of the complexity of the problem, time involved, and cost. The rapid solution of design equations by electronic computers allows the engineer to at least approach this ideal in design practice.

Many calculations required for the design of chemical and petroleum processes are based on trial and error solutions. Classic examples are those which occur in distillation calculations where many heat and material balances have to be made to obtain a design. The electrical rebalance feature of the analog computer can be used to advantage for these types of calculations without having to program a convergence to a solution. Franks and O'Brien[15] utilize this principle in performing steady-state design calculations for a multi-component, non-ideal distillation column. Consider the material balance equations for the bottom of a distillation column:

$$L_{m+1}\ \dot{x}_{m+1} = V_m y_m + W x_w \qquad (4)$$

Franks and O'Brien have developed the computer circuit shown in Figure 6 for solving this equation.

The principle of electrical rebalance performs the material balance as follows; assume that the output of the integrator driving the servo multiplier is E_o volts. The integrator will continue to integrate until the net sum of the inputs is reduced to zero and the computer comes to balance with a constant output voltage. Therefore,

$$L_{m+1}\ E_o - V_m y_m - W x_w = O \qquad (5)$$

A comparison with the material balance equation shows that E_o is now equivalent to the liquid composition **x**. Thus by supplying voltages proportional to the products $V_m y_m$, $W x_w$ and the value L, this circuit will always rebalance to give the value of liquid composition which satisfies the material balance equation. Since the design of many types of processes require the calculation of equalibrium conditions, analog circuits such as this can be extremely useful in solving chemical design problems.

Investigating reaction mechanisms are another group of problems which can be solved with the analog computer. The investigation of reaction kinetics for various proposed mechanisms is largely a trial and error procedure since a promising rate equation is assumed, and then the design calculations are carried out to see whether or not the predicted conversions agree with experimental data. By repetition of this procedure an equation for the rate can be found.

As an example consider the following example of a first order reaction carried out adiabetically. Typical equations for this reaction are:

$$-\frac{dc}{dt} = kc \qquad (6)$$

If k is given by the usual Arrhenius Equation, the equation for the rate of change of the reaction rate can be shown to be

$$\frac{dk}{dt} = \left[\frac{kE}{RT^2}\right] \frac{dT}{dt} \qquad (7)$$

This equation can be solved with the computer circuit shown in Figure 7. By changing the parameters in the reaction, the reaction curve obtained on the computer can be matched to experimental data. When the data is matched, the proper mechanism has been obtained. Should several reactions be taking place they are simultaneously represented by similar circuits. Usually a mass and heat balance is made to obtain the reaction temperature and the product produced. The net effect is to design a chemical experiment electronically to give a simulated pilot plant. Since full scale operating conditions are easily scaled into the computer as pilot plant data, the computer simulation is usually based on the actual process. This does not mean to imply that a pilot plant is no longer required. The hope is, however, that with the analog computer the use of pilot plants in process design can be put on a more rational basis.

Analog computers are also well suited to the solution of catalytic reactor design problems. Programming simplicity is maintained even when complex reactions are considered for steady state or transient cases. Wehner and Wilhelm[16] have discussed an isothermal reactor involving axial diffusion and flow with a first order

EQUATION:

$$\frac{dk}{dt} = \frac{KE}{RT^2} \frac{dT}{dt}$$

COMPUTER CIRCUIT:

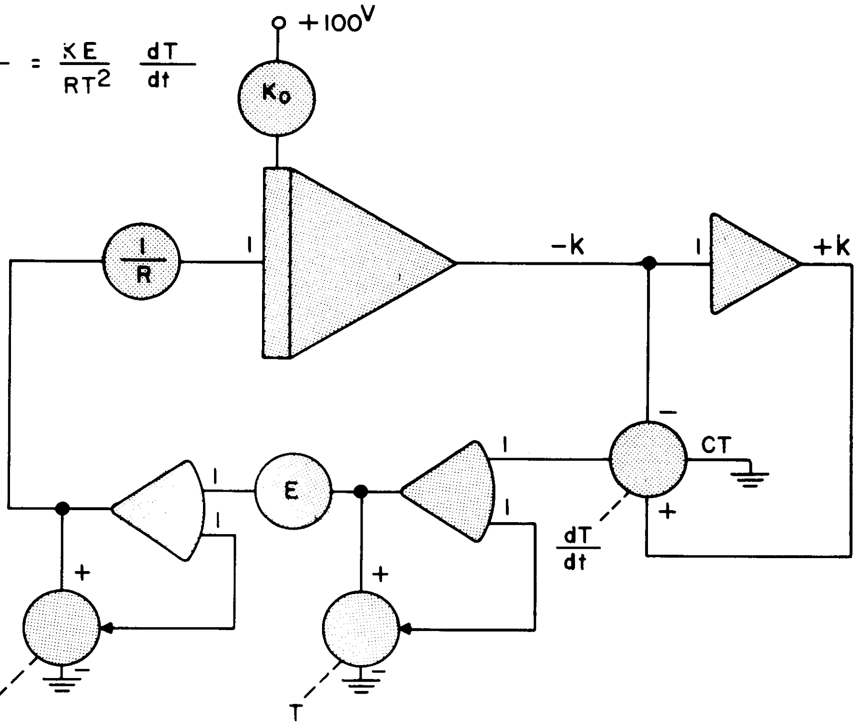
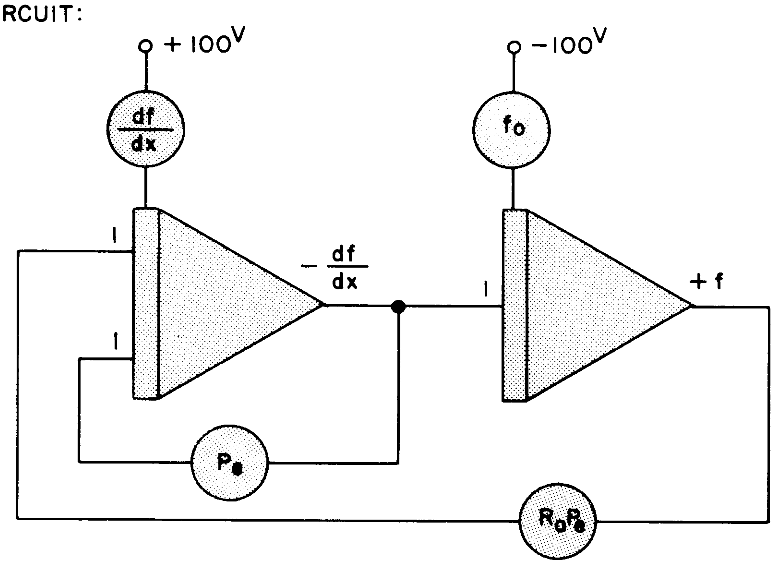


FIGURE 7. ANALOG SIMULATION FOR A FIRST ORDER CHEMICAL REACTION.

EQUATION:

$$\frac{1}{P_e} \frac{d^2 f}{dZ^2} - \frac{df}{dZ} - R_a f = 0 \qquad 0 \leq Z \leq 1$$

COMPUTER CIRCUIT:



NOTE:
$X = 1 - Z$

FIGURE 8. ANALOG SIMULATION OF SIMPLE MODEL OF A CATALYTIC REACTOR

reaction. The computer circuit for this simple model is shown in Figure 8. Note that the circuit is set up so that the integration is carried out in the opposite direction; i.e., from the back of the reactor to the front. The conclusions reached by the authors dictate this change in variable if an analog computer solution is to be obtained. This basic model may be expanded to more complicated investigations with small changes in circuitry. These are; (a) higher order reactions, (b) adiabatic reactor, (c) nonsteady state operation, and (d) consideration of radical and axial diffusion.

Because of the similarity of the basic equations describing chemical reactions, there appears to be a wide variety of chemical design problems amiable to solution by the analog computer. A partial list of possible applications are:

Classical Kinetics	Cooler Condenser
Particle Fluid Transfer	Drying
Transfer in the Flowing Liquid	Distillation
Catalysis	Absorption
Fluidized Bed Reactors	Solvent Extraction
Adiabatic Absorption	Zone Melting
Ion Exchange	

Conclusions

This discussion has attempted to point out the types of processing problems which can be solved by Analog Simulation. The use of the analog computer in the processing industries has increased tremendously during the past year and indications are that this trend will continue. Perhaps the most important reason for this increase is the enthusiastic acceptance of the analog computer as the "engineers tool" by those who have performed calculations with the machine. The simplicity of programming has allowed the engineer with the problem to conduct the investigation himself. Rapid changes in programs, which were unforseen until problem solutions were obtained, are easily made during the course of the investigation and can be guided by the engineer who knows the problem. The net result is that the engineer develops a "feeling" for the problem which helps him produce better results and often at a cost of several magnitudes less than that of other forms of computation.

Experience has shown that analog computers are highly capable tools for solving engineering problems associated with the design and control of industrial processes. Their use effectively expands the utility of the engineer. Yet they cannot think for him nor will they automatically solve his problems. In fact, considerable effort must be expanded before solutions to problems are obtained. Because of this, the use of a computer in engineering work generally does not replace any men but rather increases the amount of effective work that can be done by the same staff. One important result is that problems and methods are set up for computer solution that would never be considered without the machine. Another advantage and one that is often overlooked is that the techniques associated with the use of a computer allows problems to be classified which in itself encourages a broader investigation of possible solutions to problems. This cannot help but result in improved engineering effort.

Nomenclature

M	=	Mass of moving parts including diaphragm plates, lb/FT-sec ^{2}.
D	=	The pneumatic damping constant, lb/FT-sec.
K	=	Spring constant, lb/FT.
x	=	The travel of the Valve stem, FT.
F	=	The force of the actuating air pressure, lb.
P_o	=	Controller output pressure, PSI.
ξ	=	Controller error, dimensionless.
K_c	=	Controller gain, dimensionless.
τ_r	=	Reset time constant, seconds.
T_o	=	Thermocouple output temperature, ^{0}F.
T_i	=	Thermocouple input temperature, ^{0}F.
K_i	=	Thermocouple proportionality constant, dimensionless.
τ_1	=	Thermocouple well time constant, seconds.
τ_2	=	Thermocouple time constant, seconds.
l	=	Number of sections used to represent heat exchanger.
T_s^n	=	Temperature of vapor in nth section, ^{0}F.
T_p^n	=	Temperature of product in nth section, ^{0}F.
$T_p^{N\,1}$	=	Average temperature of product in nth section, ^{0}F.
t	=	Time in seconds.
ω_p	=	Product mass flow rate, lbs/sec.
M_p	=	Product mass, lbs.
C_p	=	Product specific heat, Btu/lb-^{0}F.
U	=	Overall heat transfer coefficient, Btu/sec.-FT^2-^{0}F.
A	=	Heat transfer area, FT^2.
L_{n+1}	=	Liquified flow from plate above, lb. mol/hr.
x_{n+1}	=	Liquid composition on plate above in mol fractions.
V_n	=	Vapor flow from bottom plate, lb. mol/hr.
y_n	=	Vapor composition on bottom plate in mol fractions.
W	=	Bottoms product take off rate, lb. mol/hr.
x_w	=	Bottoms composition in mol fraction.
k	=	Reaction rate constant.
C	=	Concentration of reactant.
E	=	Activation energy.
R_a	=	Gas constant
P_e	=	Peclet number (uL/D), dimensionless.
R	=	Reaction number (kL/u), dimensionless.
f	=	Fraction of a reactant remaining, dimensionless.
z	=	Normalized distance variable, dimensionless.
u	=	Flow velocity, length/time.
L	=	Reactor length, length.

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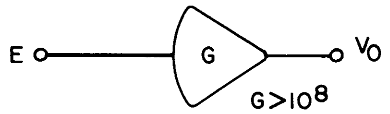
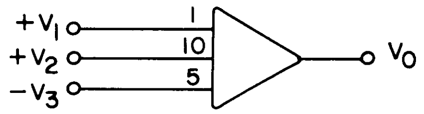
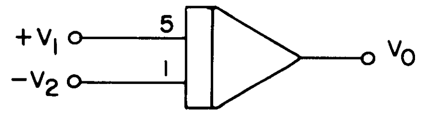
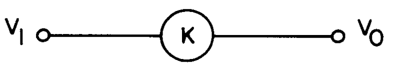
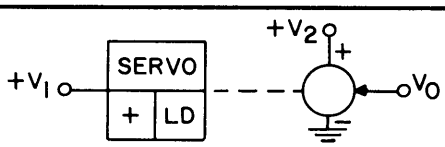
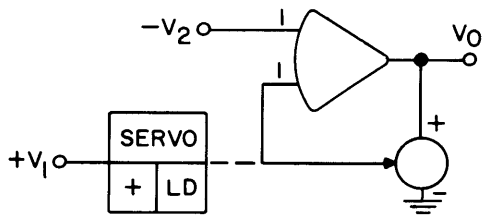
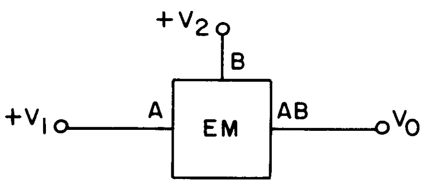
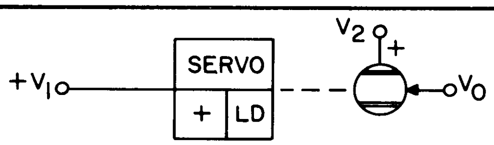
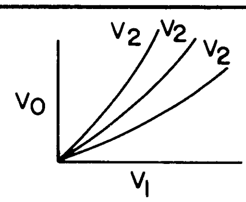
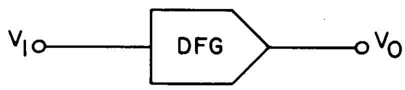
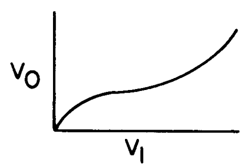
TABLE I

SYMBOLS FOR P A C E GENERAL PURPOSE ANALOG COMPUTER

NAME	SYMBOL	FUNCTION	DESCRIPTION
HIGH GAIN AMPLIFIER		$V_O = -GE$	OPERATIONAL AMPLIFIER
SUMMER		$V_O = -(V_1 + 10V_2 - 5V_3)$	AMPLIFIER MULTIPLE INPUT
INTEGRATOR		$V_O = -\int (5V_1 - V_2)\,dt$	AMPLIFIER MULTIPLE INPUT
COEFFICIENT POTENTIOMETER		$V_O = KV_1$ $0 < K < 1$	MANUALLY SET POTENTIOMETER
SERVO MULTIPLIER		$V_O = +\frac{V_1 V_2}{100}$	SERVO DRIVEN POTENTIOMETER
DIVISION CIRCUIT		$V_O = +\frac{100\,V_2}{V_1}$	HIGH GAIN AMPLIFIER AND SERVO DRIVEN POTENTIOMETER
ELECTRONIC MULTIPLIER		$V_O = -\frac{V_1 V_2}{100}$	ELECTRONIC MULTIPLIER
SERVO FUNCTION GENERATOR			ARBITRARY FUNCTIONS
DIODE FUNCTION GENERATOR			

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