HITACHI Analog•Hybrid Computer

Technical Information Series No. 3

Double Integral

《Calculation of Volume of Cone》

1 9 6 7

Hitachi, Ltd.

printed in Japan

The analog computer can provide a pattern display which presents the solution in a visual figure such as the three-dimensional representation of a volume.

This is a districtive advantage from the human engineering point of view, and this is the reason why a problem of double integral has been selected for this application manual.

The computation of the volume of cone is described below, in which one half of the volume is calculated and the right solution shall be,

$$V_h = \frac{1}{2} \times \frac{\pi r^2 h}{3} \quad \text{(or, with } r = h = 1\text{)}$$

$$V_h = 0.502$$

With an analog computer, pattern displays by high speed computation is possible, which is almost impossible with an digital computer, although a solution by an analog computer may contain an error of approximately 0.2%. *

*Note: This error is produced by non-linear operations such as calculations of square and square root.

In this example, the analog computer is used in a low speed operation in order that the solutions can be given by a pen recorder as well as by a pattern display. However, pattern displays as many as 100 to 1,000 times per second can be obtained if the reset and the compute time is controlled by the oscilloscope, the timer or the logic elements.

With a modification on the blockdiagram given in this example, a pattern display can be easily obtained in which the radius "r" of the base of the cone or the height "h" of the cone is chosen as the parameter.

This example illustrates the following three of the various features of HITACHI 505.

1. Independent control of each integrator.
2. Operation on multiple time axes.
3. Pattern displays.

Example of Analysis - Double Integral
The volume of a cone is pattern-displayed by means of multiple time axes operations.

1. Equation

With the coordinates selected as shown in Figure 1, the equation of a cone is,

$$x^2 + y^2 = k^2 z^2 \quad \cdots\cdots \quad (1)$$

$$k = \frac{r}{h} \quad \cdots\cdots\cdots\cdots\cdots \quad (2)$$

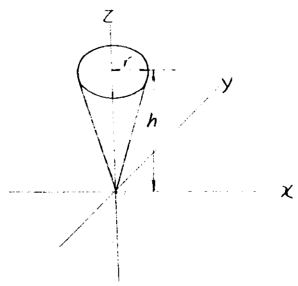


Figure 1

where,

h: height of the cone

r: radius of the base of the cone

From equation (1),

$$y = \pm\sqrt{k^2z^2 - x^2} \quad \ldots\ldots (3)$$

In equation (3), the height of the cone in reference to x - z plane is given.

Since the cone is symmetrical to x - z plane, the volume of the cone on one side of the plane (for example, in the extent $y \geq 0$) is one half the total volume V. In this example, the calculation is performed for $y \geq 0$.

The area S_h of a cross section parallel to the base, for the half of the cone ($y \geq 0$), is given by the following equations.

$$S_h = \int_{-kz}^{kz} \sqrt{k^2z^2 - x^2}\, dx \quad \ldots\ldots\ldots (4)$$

For the calculation of the volume of the cone, the cone is sliced at equal intervals by planes parallel to the base of the cone.

Each section of the cone between two adjacent planes is approximated by a cylinder, the volume of each cylinder is calculated, and the volume of the cone is obtained as the sum of those of the cylinders.

To apply this method of calculation to the half ($y \geq 0$) of the cone, we have the following equations for the volume of a section, $\triangle V_h$.

$$\triangle V_h = \triangle Z \cdot S_h = \triangle Z \int_{-kz}^{kz} \sqrt{k^2z^2 - x^2}\, dx \quad \ldots\ldots (5)$$

The total volume V_h of the half of the cone, with the number of sections N, is given by the following equation.

$$V_h \doteqdot \sum_{n=0}^{N-1} \triangle V_h = \sum_{n=0}^{N-1} \triangle Z \int_{-kz}^{kz} \sqrt{k^2z^2 - x^2}\, dx \quad \ldots\ldots (6)$$

The true volume can be obtained by an infinite number of divisions, or

$$V_h = \lim_{n \to \infty} \sum_{n=0}^{N-1} \triangle Z \int_{-kz}^{kz} \sqrt{k^2z^2 - x^2}\, dx$$

$$= \int_0^z dz \int_{-kz}^{kz} \sqrt{k^2z^2 - x^2}\, dx \quad \ldots\ldots\ldots\ldots\ldots (7)$$

This is, the true volume is obtained by a double integral calculus. In an analog computer, the calculation with $N \to \infty$ can not be performed. Or strictly speaking, equation (6) is solved in an analog computer.

2. Equations for the calculation in the computer

To simplify the problem, the calculation of the volume of a cone with r = h = 1 is considered.

From equation (2), it follows that $k = \frac{r}{h} = 1$.

Equation (6) can be expressed as,

$$V_h \doteqdot \sum_{n=0}^{N-1} \triangle z \int_{-z}^{z} \sqrt{z^2 - x^2}\, dx \ldots\ldots\ldots\ldots\ (8)$$

The scales of the variables are converted on the basis that an integration with x is performed in 0.2 second and the summation is done in 100 seconds. The scales are shown in the following table.

Variable inequation	Assumed maximum value	Scale factor	Computer variable
x	± 1	±10 time/length	$\left[10t\right]$
z	1	$\frac{1}{100}$ time/length	$\left[\frac{To}{100}\right]$
V_h	1	1/volume	$\left[V_h\right]$

Equation (8) expressed in terms of the computer variables is,

$$\left[V_h\right] \doteqdot \sum_{n=0}^{N-1} \frac{z}{N} \int_{-z}^{z} \sqrt{z^2 - x^2}\quad dx$$

$$= \sum_{n=0}^{N-1} \left[\frac{To}{100}\right] \frac{1}{N} \int_{-\frac{T}{100}}^{\frac{T}{100}} \sqrt{\left[\frac{T}{100}\right]^2 - \left[10t\right]^2}\ \left[10\ dt\right]$$

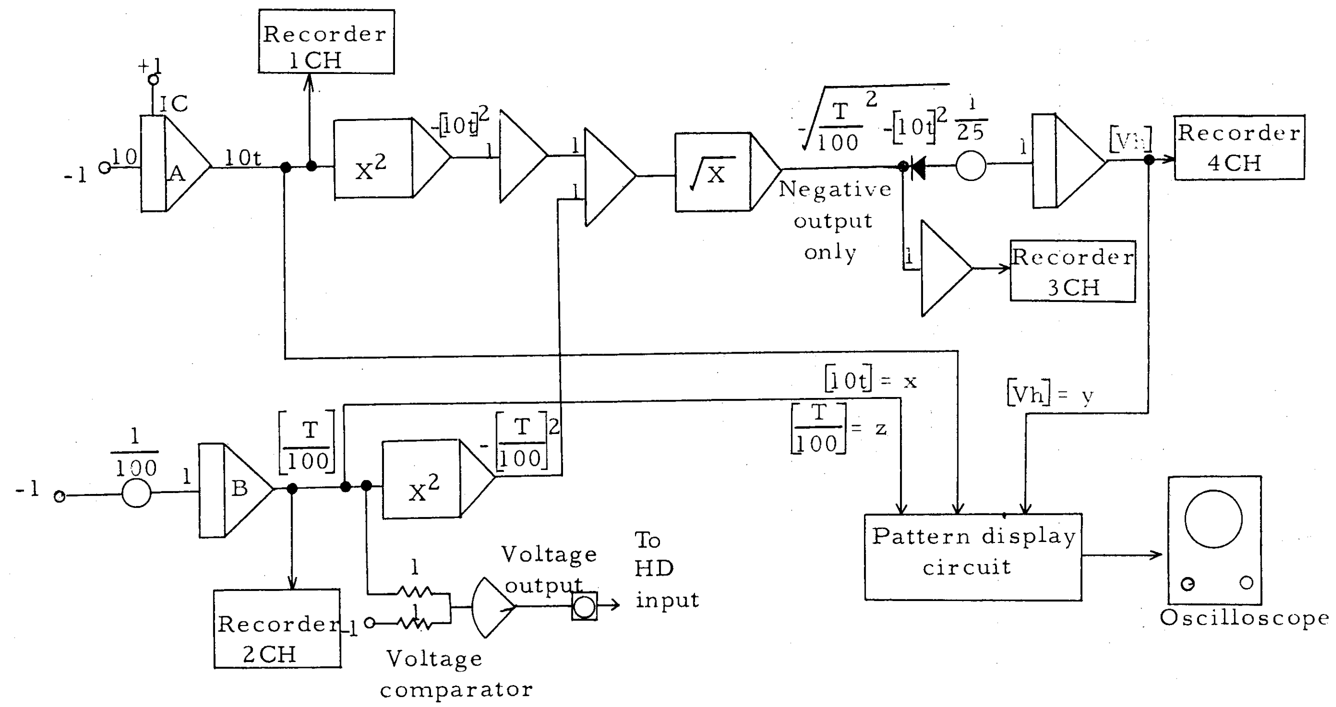
$$= \sum_{n=0}^{N-1} \frac{To}{10N} \int_{-\frac{T}{100}}^{\frac{T}{100}} \sqrt{\left[\frac{T}{100}\right]^2 - \left[10t\right]^2}\ dt \ldots\ (9)$$

For the calculation time To = 100 seconds, and the number of divisions N = 250, equation (9) is,

$$\left[V_h\right] \doteqdot \sum_{n=0}^{249} \frac{1}{25} \int_{-\frac{T}{100}}^{\frac{T}{100}} \sqrt{\left[\frac{T}{100}\right]^2 - \left[10t\right]^2}\quad dt \ldots\ldots\ (10)$$

Equation (10) is calculated by the analog computer.

3. Blockdiagram



RS; RESET

CP; COMPUTE

HD; HOLD

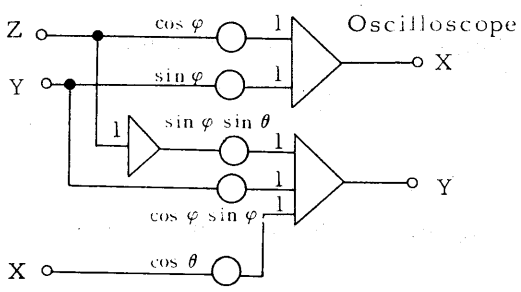
AR; ALL RESET

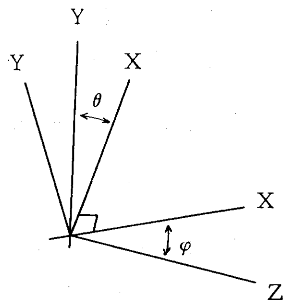
A ; Group A

B ; Group B

Integrator \ Number of operations	0	1	2	3 - 500	501
Group A	AR	CP 0.2 seconds	CP 0.2 sec.	Repetition of CP and RS	HD
Group B	AR	CP for 100 seconds			HD

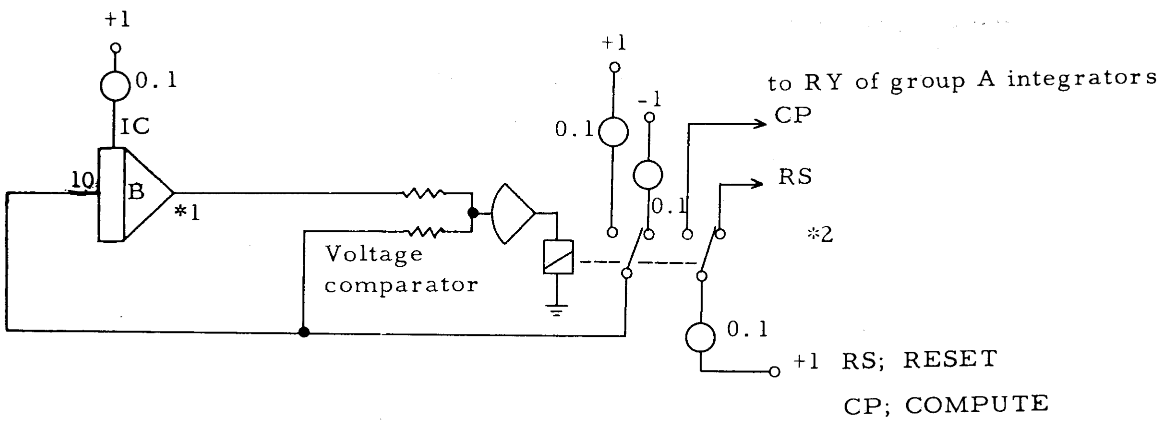
Pattern display circuit



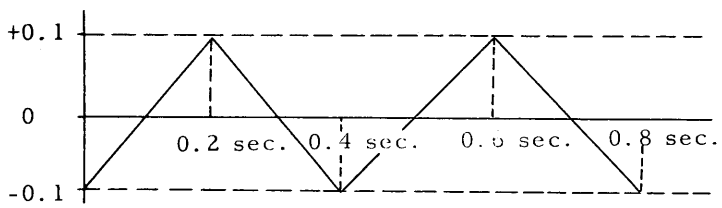


This is a conversion circuit for displaying a three dimensional figure on X-Y axes, rotating x, y, and z axes by $\theta°$ and $\varphi°$

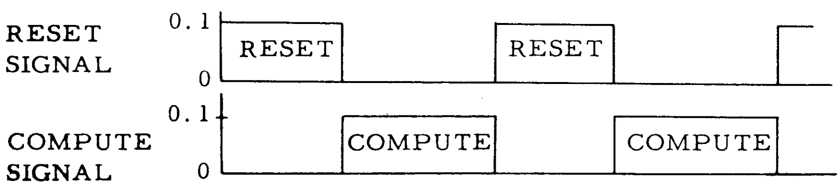
Circuit generating RS and CP time of 0.2 seconds.



*1 The output of the integrator is shown in the figure below.



*2 Reset and compute signals in the figure below are generated by the relay contact.



4. Computing elements required

Dual DC amplifier;

7 units (Of 14 Amplifiers; 13 employed in the computation.)

DA-151 or DA-151A

Dual Integrator;

2 units (Of 4 Integrators; all employed in the computations.)

IN-151 or IN-153

Potentiometer panel;

1 panel (of 18 Potentiometers; 11 employed in the computation.)

PT-251

Potentiometer patching units;

3 units PT-151

Voltage comparators;

1 unit (of 4 Comparators; 2 employed in the computation)

CP-151, CP-152 or CP-153

Square function generator;

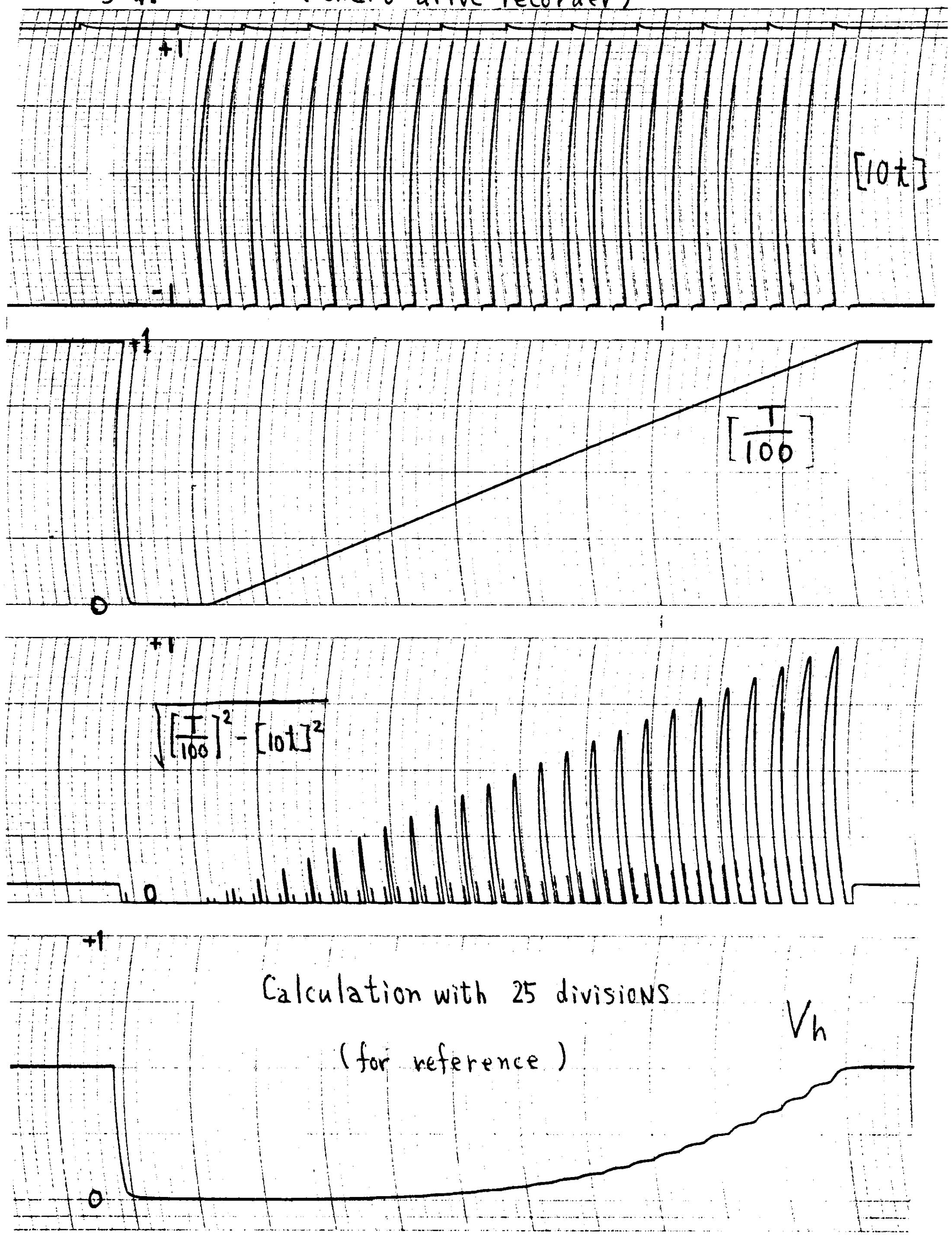
1 unit (of 4 Function generators; 3 employed in the computation.)

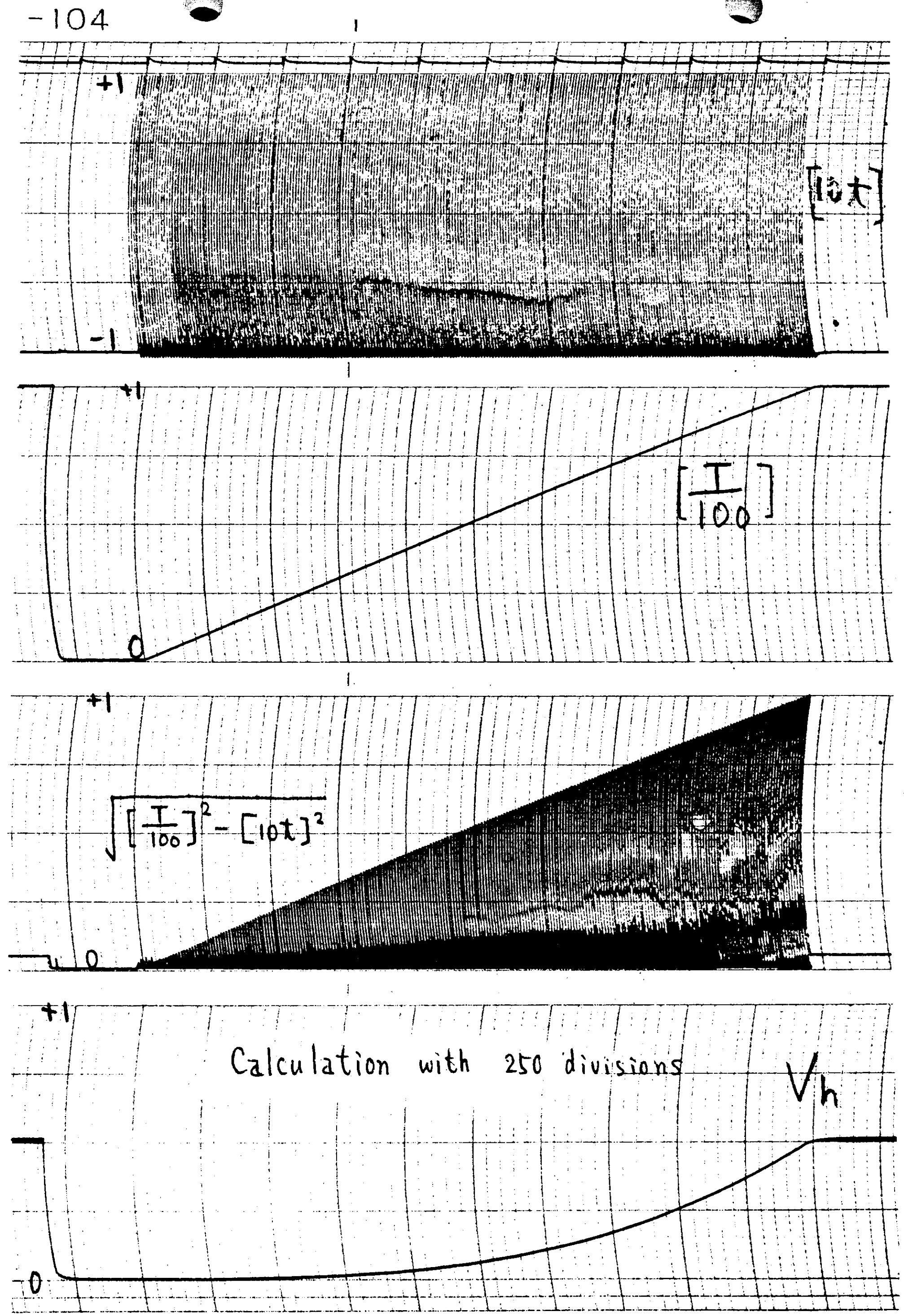
FG-154A

5-1 Solution (Chart of pen recorder) Page 7

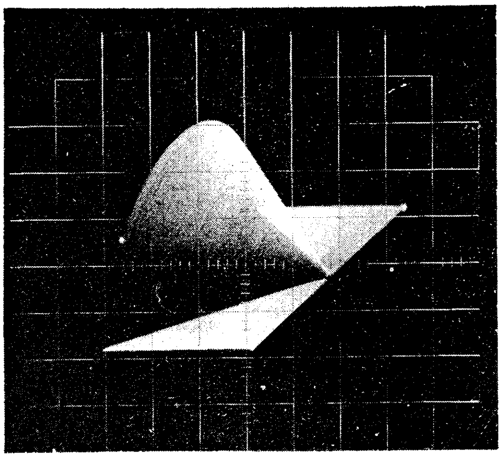
5-2 Solution (Display on oscilloscope) Page 8

5-1. Solution (chart drive recorder)



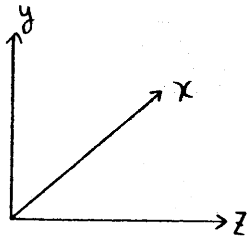


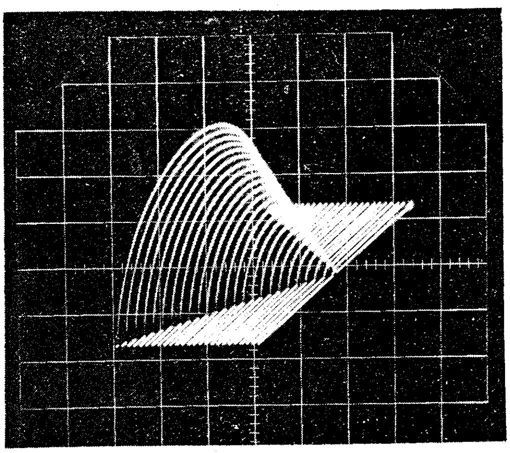
5-2 Solution (Pattern display)



Example of calculation with

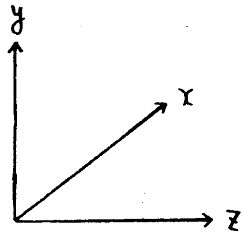
N = 250

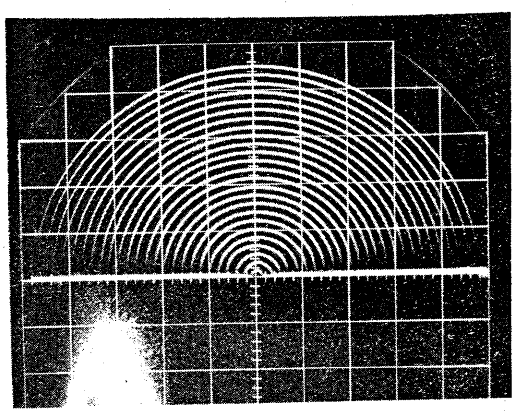




Example of calculation with

N = 25 (for reference)

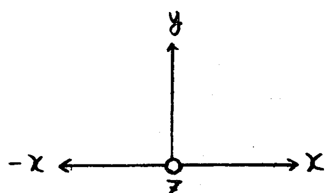




Output of $\sqrt{z^2 - x^2}$ with

N = 25

where $z^2 - x^2 \geqq 0$ (for reference).



Z axis is perpendicular to the picture.